Proof Writing Tips

Janet Leahy (created for CPSC 413 in Winter 2021)

with minor modifications by Renate Scheidler

January 14, 2023

First, some shorter points of advice.

- State definitions when you use them. This is important for clarity, and it will help improve your chances for partial marks.

- When working with inequalities, make sure you are applying them in the right direction. For example, if you know $x > y$ and $50 > y$, you can't conclude that $50 > x$ (try it with $x = 100$ and $y = 10$).

- Have you considered all possible cases? A common error is that the proof you give doesn't work for border cases, such as a graph with no edges, or $x = 0$, etc. If the proof you have automatically handles these cases, then don't add anything extra, but make sure to check that it does.

- It is very much worth your time to double-check your proof for typesetting, spelling, and grammar errors. Make sure you write in complete sentences. I can't stress enough how important it is to explain your idea well. A good idea that is poorly explained might not get conveyed, and then you won't get the credit you deserve for it.

- If you don't know how to prove something, explain what you do know and what you have figured out, and point out where you got stuck. This will almost always give you more partial marks than pretending you do know and writing a non-proof. Also, if you explain what you understand and what you don't, the feedback you get will be more useful for figuring out how to deal with similar problems in the future.

- Learning to write good proofs takes time and practice. There isn't a shortcut on this, so be patient and pay attention to the feedback you get. If you are feeling stuck, the best strategy is to read example proofs wherever you can find them, and try to understand why they are written the way they are.

- At the end of the day, it doesn't matter whether your proof uses induction, contradiction, construction, and so on. These are just standard templates that people use to write proofs because they are often easy to work with (and which particular one you use can just be a matter of personal taste). All that ultimately counts is that your proof is clear, thorough, correct, and well-presented.

The following deserves more than a bullet point, as it is both one of the most serious mistakes I see when marking, and one of the more common ones too:

Do **NOT** assume what you want to prove, and then "prove" it by deriving something true. Logically, this doesn't make sense, and is not a valid proof.

For example, consider the following argument:

(1) Suppose penguins can fly.
(2) If something can fly, then it has wings.[1]
(3) Therefore penguins have wings.
(4) This is true, therefore penguins can fly

As you can see, the reasoning for steps 1-3 is sound, but we "proved" something incorrect. This is because of the flaw with the proof structure. The way this usually manifests in proofs I see is something like the following:

Statement: Prove that $A < B$ when [some condition] (for some expressions A and B).

Proof:

$$A < B$$
$$A - 1 < B - 1$$
$$\sqrt{A - 1} < \sqrt{B - 1}$$
$$\ldots$$

Some true statement

Again, this is not a good way to write a proof. It can be technically correct if each of the statements is an if-and-only-if statement, but is not considered a good structure and there is a signicant chance you will have some major flaw in your logic.

What you should do instead is flip the order of statements. If you start with [Some true statement], write the statements in the opposite order (making sure each one is implied by the previous one), and finish with $A < B$, that counts as a valid proof.

By trying to do this to the penguin argument, we can see that it breaks:

(1) Penguins have wings
(2) If something can fly, then it has wings
(3) ?

Here, we get stuck, because (2) does not say that if something has wings, then it can fly (it is possible to have wings and not be able to fly, such as with penguins).

In conclusion, it is fine to start with what you are trying to prove and work with that when you are initially figuring out how to tackle the problem. However, when you write up the final proof, do not start by writing what you want to prove and deriving a true statement from that. One way to fix such a proof is to start with the true statement and work backwards so that the last thing you write is the statement you want to prove.

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[1]Pretend this is a true fact.

Addendum. "Suppose" is a loaded word when writing proofs, so use it well. Whenever you write it, you should be able to replace it with the phrase "Assume for now (and I'll justify later)". It has the same effect as "if", but can apply for longer than a single sentence. For example, the following short proof is an example of how to properly use "suppose".

Statement:
$$f(x) = \begin{cases} 2x & \text{if } x \geq 5, \\ 0 & \text{otherwise.} \end{cases}$$

Prove that $f(x)$ is even for all $x \in \mathbb{Z}$.

Proof: First, suppose $x \geq 5$. Then $f(x) = 2x$, which is even. Next, suppose $x < 5$. Then $f(x) = 0$, which is even.[2] Either $x \geq 5$ or $x < 5$, so this shows that $f(x)$ is even for all $x \in \mathbb{Z}$.

Note that I could have just written "If $x \geq 5$, then $f(x) = 2x$, which is even". For more complicated proofs, you will need "suppose", because you won't be able to cram the entire case into one "if" sentence. Also notice the justification for the supposing in the last sentence.

Finally, as indicated in the footnote, do NOT "suppose" things that are either given to you in the question, or that are known to be true. Just state these things as facts. The purpose of "suppose" is to replace an "if" that is too long to fit in a sentence. End of rant.

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[2]Never say, for example, "suppose 0 is even". This is a fact, so you don't need to suppose it.